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Some existence results for impulsive nonlinear fractional differential equations with mixed boundary conditions

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ABSTRACT

This paper investigates the existence and uniqueness of solutions for an impulsive mixed boundary value problem of nonlinear differential equations of fractional order $\alpha \in (1, 2]$. Our results are based on some standard fixed point theorems. Some examples are presented to illustrate the main results.

1. Introduction

Boundary value problems for nonlinear fractional differential equations have recently been addressed by several researchers. The interest in the study of differential equations of fractional order lies in the fact that fractional derivatives provide an excellent tool for the description of memory and hereditary properties of various materials and processes. With this advantage, the fractional-order models become more realistic and practical than the classical integer-order models, in which such effects are not taken into account. As a matter of fact, fractional differential equations arise in many engineering and scientific disciplines such as physics, chemistry, biology, economics, control theory, signal and image processing, biophysics, blood flow phenomena, aerodynamics, fitting of experimental data, etc. [1–4]. For some recent development on the topic, see [5–19] and the references therein.

Impulsive differential equations, which provide a natural description of observed evolution processes, are regarded as important mathematical tools for the better understanding of several real world problems in applied sciences. The theory of impulsive differential equations of integer order has found extensive applications in realistic mathematical modelling of a wide variety of practical situations and has emerged as an important area of investigation in recent years. For the general theory and applications of impulsive differential equations, we refer the reader to the references [20–23]. On the other hand, the impulsive boundary value problems for nonlinear fractional differential equations have not been addressed so extensively and many aspect of these problems are yet to be explored. For some recent work on impulsive differential equations of fractional order, see [24–31] and the references therein.

In this paper, we investigate the existence and uniqueness of solutions for a mixed boundary value problem of nonlinear impulsive differential equations of fractional order $\alpha \in (1, 2]$ given by

$$\begin{cases} {}^{C}D^{\alpha}u(t) = f(t, u(t)), \quad 1 < \alpha \leq 2, t \in J', \\ \triangle u(t_k) = I_k(u(t_k)), \qquad \triangle u'(t_k) = I_k^*(u(t_k)), \quad k = 1, 2, \ldots, p, \\ Tu'(0) = -au(0) - bu(T), \qquad Tu'(T) = cu(0) + du(T), \quad a, b, c, d \in \mathbb{R}, \end{cases} \tag{1.1}$$

where ${}^{C}D^{\alpha}$ is the Caputo fractional derivative, $f \in C(J \times \mathbb{R}, \mathbb{R})$, $I_k, I_k^* \in C(\mathbb{R}, \mathbb{R})$, $J = [0, T](T > 0)$, $0 = t_0 < t_1 < \cdots < t_k < \cdots < t_p < t_{p+1} = T$, $J' = J \setminus \{t_1, t_2, \ldots, t_p\}$, $\triangle u(t_k) = u(t_k^+) - u(t_k^-)$, $u(t_k^+)$ and $u(t_k^-)$ denote the right and the left limits of $u(t)$ at $t = t_k (k = 1, 2, \ldots, p)$, respectively and $\triangle u'(t_k)$ have a similar meaning for $u'(t)$.

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