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Fixed point theorems for convex contraction mappings on cone metric spaces

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ABSTRACT

In 2007, Huang and Zhang [L.G. Huang, X. Zhang, Cone metric spaces and fixed point theorems of contractive mappings, J. Math. Anal. Appl. 332 (2007) 1468–1476] rediscovered normal cone metric spaces and obtained the Banach contraction principle for this setting. Later on, Rezapour and Hamlbarani [Sh. Rezapour, R. Hamlbarani, Some notes on the paper: Cone metric spaces and fixed point theorems of contractive mappings, J. Math. Anal. Appl. 345 (2008) 719–724] showed that there are non-normal cones and that the assumption of normality is redundant.

In this paper, we obtain a generalization of the Banach contraction principle to the class of convex contractions on non-normal cone metric spaces. Our result includes, as special cases, the recent results of Huang and Zhang (2007) [2] and Rezapour and Hamlbarani (2008) [3].

1. Introduction and Preliminaries

In 1922, Banach proved the following famous fixed point theorem [1]: Let (X, d) be a complete metric space and let T be a contraction mapping on X, i.e., there exists $k \in [0, 1)$ satisfying

$$d(Tx, Ty) \leq kd(x, y) \quad \text{for all } x, y \in X.$$

Then there exists a unique fixed point $z \in X$ of T. This theorem is known as the Banach contraction principle and is a forceful tool in nonlinear analysis. This principle has many applications and has been extended by several authors.

Recently, Huang and Zhang [2] generalized the Banach contraction principle to normal cone metric spaces, replacing the set of real numbers by an ordered Banach space in the definition of metric, and obtain some fixed point theorems for contractive type mappings. Since then, there have appeared many papers containing interesting fixed point results in cone metric spaces. Most of those papers deal with fixed point results in normal cone metric space. Rezapour and Hamlbarani [3] extended the results of Huang and Zhang to non-normal cone metric spaces.

In this paper, we generalize the Banach contraction principle to the class of convex contractions on non-normal cone metric spaces. Our result includes, as special cases, the main results of [2,3] and also, extend the work of Istrăţescu [4] to cone metric spaces.

Let E be a real Banach space and P a subset of E. P is called a cone if and only if

(i) P is closed, nonempty and $P \neq \{\theta\}$;

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